One of the most important tasks in solving problems of control technical systems is the evolutionary behavior formalization of the object. These models are required for the formation control algorithms. The engineers operate only with the basic linear models, leaving the solution of problems of regulation of nonlinear phenomena in industrial controllers. As a result, there are actually functioning control systems that work not only optimal, but, even in manual mode set up control processes.

Modeling of dynamical system consists of a selection of three components: i) the definition of phase space in terms of restrictions; ii) selection of discrete or continuous time, and iii) the law of evolution — the mapping of any given point in phase space and any time value in one uniquely a certain state of the system. For the discrete time evolution law is sought in the forms:

\[ x(t + 1) = \psi(x, t, u), \]
\[ y = h(x(t)), \quad t \in \mathbb{Z}, \]

for continuous time:

\[ \dot{x} = f(x, u, t), \]
\[ y = \tilde{h}(x(t)), \quad t \in \mathbb{R}, \]

where \( x \) — state vector; \( y \) — measured processes (hereinafter, if it is not defined features of the problem, we assume that monitoring is available, all the states); \( t \) — time; \( f, \psi, h, \tilde{h} \) — continuous and smooth vector functions.

The simulation method presented in this chapter is to construct, based on studies of experimental data, equations, whose solution has an adequate dynamic behavior modeling in this qualitative dynamic geometry of the process.

The first work on the reconstruction of the strange attractor from the time series has been publishing the results on hydrodynamics [1]. The article shows that you can get a satisfactory picture of the strange attractor of the geometric dimensions of a small, if the variables \( x \), appearing in the equations of the dynamical system \( \frac{dx}{dt} = F(x) \), use the m-dimensional vectors, derived from the
elements of time series on the same principle, that in the problems of autoregression. That same year, F. Takens reported on his theorem, which was published a year later [2]. That it is the basis of all algorithms for time series analysis methods of nonlinear dynamics. The problem of determining the form of a dynamical system from its one-dimensional realization belongs to a class of incorrect problems. Unlike the problem of analyzing this issue is ambiguous, since there are infinitely many dynamical systems of various kinds can play the existing signal with a given degree of accuracy.

The method of the global reconstruction of a dynamic system of equations for its one-dimensional realization was proposed in [3, 4]. The algorithm is as follows. One-dimensional realization of the process in a system, which is considered a "black box" recovered phase portrait on the Takens theorem, topologically equivalent to the attractor of the original system. According to a priori given equations, is the method of least squares a set of unknown coefficients

Now there is considerable amount of publications, developing and constantly improving the proposed the method [5–11]. For example, in R. Brown and others [12] to reconstruction dynamic equations on the experimental time series with a broadband continuous spectrum use additional information about the dynamic and statistical properties of the original system contained in the implementation. In obtaining equation takes into account the values of Lyapunov exponents and the probability density, calculated from the original time series. However, the resulting evolution equations have a very cumbersome, inconvenient to use. In [13] used the hidden variables to write model equation. In [12] describes a method for synchronizing the model with the original data. In a number of O.L. Anosov proposed reconstruction algorithm scalar differential equations for systems with delay.

However, the feature of many studies is that the proposed methods are illustrated with examples of simple low-dimensional model systems when we know in advance what should be the result of global reconstruction. It does not show substantial benefits given by the modifications of the basic method [3]. Described in the publications of the algorithms are tested on a number of known model systems with small dimension and a simple form right sides. The efficiency of the method is demonstrated by the example of time series generated by the real "black boxes".

The challenge is the need to work with noisy data in the processing of experimental time series. On the one hand, more desirable is the use of sequential differentiation to restore the phase trajectory, because it can get a model that
contains, in general, approximately $n$ times smaller than the coefficients of the various non-linear than when using the method of delays. But differentiation will inevitably lead to increased noise components of high order. Without pre-filtering the time dependence of the second derivative can be noise-like process. In addition, methods of attachment are obvious flaws in the analysis significantly heterogeneous implementations, i.e., signals in which areas with fast motion alternating with areas of slow motions.

Arbitrary choice of nonlinearities, as a rule, does not allow for a successful reconstruction of the dynamic equations for real systems. In particular, in [14] indicated the presence of three typical cases:

1. Reconstruction of locally describe the phase trajectory of the original system. In this case, the reconstructed model is unstable in the sense that the solution of these equations reproduces the signal under investigation only in a short period of time.

2. There is poor local predictability of the phase trajectory, but there is visual similarity of the phase portraits. Reconstruction of solution the stable in the sense of Poisson. In this case, the attractor of the reconstructed model has metric characteristics similar to those of the original attractor.

3. There is a good local predictability of the phase trajectory of any point in time values exceeding the characteristic correlation time. The phase portrait reconstructed model is identical to the original, and the system is Poisson stable.

According to [15] on centrally-stable manifold, the system around an equilibrium point $O$ takes the form

$$\begin{align*}
\dot{y} &= By + f_1(x, y, z), \\
\dot{z} &= Cz + f_2(x, y, z), \\
\dot{x} &= Ax + \psi_0(x, y, z),
\end{align*}$$

(1)

where $x \in \mathbb{R}^m$, $y \in \mathbb{R}^k$, $z \in \mathbb{R}^{n-m-k}$ ($j = \overline{1,m}$); $\text{spectr } A = \{\lambda_{m+1}, \ldots, \lambda_k\}$, $\Re \lambda_j < 0$, ($j = \overline{m+1,k}$); $\text{spectr } C = \{\lambda_{k+1}, \ldots, \lambda_n\}$, $\Re \lambda_j > 0$, ($j = \overline{k+1,n}$); $C'$-functions $f_1$, $f_2$ and $\psi_0$ together with its first derivatives vanish at the origin.

In this case, the right side of the system may depend on the control $u$ either continuously (in this case, smooth manifolds, discussed below, depend continuously on $u$), or smooth. In the latter case, $u$ is included in the number of "central" variables $x$ and, thus, further investigated the variety and the foliation will have the smoothness of $u$, equal to the smoothness of $x$. 
Theorem 1. In a small neighborhood of the equilibrium state there exists a \((m + k)\)-dimensional center-stable invariant manifold. Central-stable manifold is not uniquely defined, but for any two manifolds \(W^s_1\) and \(W^s_2\) functions \(\phi^s_1\) and \(\phi^s_2\) the class \(C^r\) that contains the point \(O\) and concerns at this point subspace \(\{z = 0\}\), define the same the same symmetries group of the point \(O\).

Theorem 2. In a small neighborhood of the equilibrium state \(O\) there exists an \((n - k)\)-dimensional invariant manifold is a \(C^r\)-smooth. Center unstable manifold includes all trajectories that remain in a small neighborhood of \(O\) for all negative values of time. For any two \(W^u_1\) and \(W^u_2\) functions \(\phi^u_1\) and \(\phi^u_2\), the class \(C^r\) containing the point \(O\) and tangent at this point subspace \(\{y = 0\}\) define the same symmetries group.

Intersection of center-stable and unstable manifolds of a central \(C^r\)-smooth \(m\)-dimensional invariant center manifold, defined by the equation of the form \((y, z) = \phi^C(x)\). Function \(\phi^C\) with all derivatives, in particular, the Taylor expansion of functions \(\phi^C\) at \(O\) is uniquely determined by the system.

Straightening of the central stable and center-unstable manifolds, as well as rectification of the strong stable and strong unstable invariant foliations on these manifolds leads to the good fact.

Theorem 3 [15]. With a \(C^{r-1}\)-smooth transformation of the system (1) can locally be reduced to

\[
\begin{align*}
\dot{y} &= \left( A + F_1(x, y, z) \right) y, \\
\dot{z} &= \left( C + F_2(x, y, z) \right) z, \\
\dot{x} &= Bx + \Psi_0(x) + \Psi_1(x, y, z)y + \Psi_2(x, y, z)z,
\end{align*}
\]

where \(\Psi_0\) — is a \(C^r\)-smooth function, which together with its first derivative vanishes at \(x = 0\); \(F_{1,2}\) are functions that vanish at the origin: \(\Psi_{1,2} \in C^{r-1}\); \(\Psi_1(x, y, 0) = \Psi_2(x, 0, z) = 0\).

Here, the local center-unstable manifold given by the equation \(\{y = 0\}\), the local center-stable manifold — \(\{z = 0\}\), and a local center manifold — \(\{y = 0, z = 0\}\). Strong stable foliation consists of surfaces \(\{x = \text{const}, z = 0\}\), and the layers are strongly unstable foliation of the form \(\{x = \text{const}, y = 0\}\).

A similar theory is constructed for discrete systems.

For systems that admit the group of symmetries and construct models of systems, reduced to a central invariant manifold results can be summarized as follows [16].
In the local region of the qualitative dynamic behavior of systems typologically equivalent system, reduced to the central manifold

\[ \dot{x} = Ax(t) + \Psi_0(x), \]

where \( \Psi_0(x) \) — is a \( C^r \)-smooth function \( (\Psi_0(x_0) = \Psi_0'(x_0) = 0, x_0 = 0) \), whose structure is determined by the symmetry transformation.

For a discrete system, the local center manifold is determined by the system:

\[ x(t + 1) = Ax(t) + \theta_0(x), \]

where \( \theta_0(x) \) — is a \( C^r \)-smooth function \( (\theta_0(x_0) = \theta_0'(x_0) = 0, x_0 = 0) \) defined on the basis of symmetry groups constructed on the reconstructed attractor.

For the reconstruction of a nonlinear system in the form (2) (3) proposed the allocation of local regions of phase trajectories \( \varphi_1(x) \) and \( \varphi_2(x) \) that are close to periodic, and the construction of finite-transformations transform one area to another. That is, the construction of the symmetry group of phase trajectories, which is characterized by the transformation of graphs:

\[ \text{graph} \{ \varphi_1(x) \} \rightarrow \text{graph} \{ \varphi_2(x) \}. \]

The resulting transformations determine the structure of the desired evolution equations.

The result of reconstruction algorithms is a set of points belonging to the attractor of the system. The task is an automatic search for symmetries of the local sections of the phase trajectories. We seek the symmetry of translation, rotation, stretching and compression.

**EXAMPLE 1. Reconstruction of the model to experimental data**

Original data set consists of 200 points. The data represent the traffic network.

![Fig. 1. Labeled attractor.](image)
The presence of the symmetry of the system determined the structure of the form (3). Parameter identification system using the method of least squares, gives the following result:

\[
A = \begin{bmatrix}
0.9413 & -0.1805 & 0.1164 & -0.0295 \\
-0.0545 & 0.8226 & 0.1622 & 0.1056 \\
0.0014 & -0.0105 & -0.4455 & 0.8471 \\
-0.0062 & 0.0341 & -0.8860 & -0.5404
\end{bmatrix},
\]

\[
\Psi_0 = \begin{bmatrix}
0.0399 \\
0.0463 \\
-0.4848 \\
-0.1851
\end{bmatrix}(\exp(t^{0.0001})\sin(t^{0.4})),
\]

\[
C = 10^4\begin{bmatrix}
2.1037 & -0.0124 & 0.1202 & -0.0302
\end{bmatrix}.
\]

Fig. 2 shows a comparison of the dynamics of the original system and the reconstructed model.

Note that in determining the conversion makes sense to consider the conservation of structural stability for flows. Based on the equivalence of all the disturbances in model assumed that the local topological equivalence should preserve the structural stability, and the conversion can be close enough to the identity for small perturbations.
The use of symmetric properties of evolution equations can be successfully used for the reconstruction of the system. Developed a constructive method for constructing evolutionary models based on geometric analysis of the experimentally obtained phase portrait of the system.

This work was written by funding of grant RFBR N. 11-07-00772-a.

REFERENCES